```
SetOptions[
  {
   Plot,
   ListLinePlot, ParametricPlot, ListPlot, ListLogPlot, Plot3D,
   ListLinePlot
  },
  AxesStyle -> Directive[
    Hue[5 / 6],
    8
   ]
 ];

f[x_] := Sin[Exp[4 * x]]

φ0[x_] := Sin[10 x]; φ1[x_] := Sin[20 x];
φ2[x_] := Sin[30 x]; φ3[x_] := Sin[40 x]; φ4[x_] := Sin[50 x];

A00 = NIntegrate[φ0[x]^2, {x, 0, 1}]

0.477176

A01 = NIntegrate[φ0[x] φ1[x], {x, 0, 1}]

-0.0107339

A02 = NIntegrate[φ0[x] φ2[x], {x, 0, 1}]

0.0135097

A03 = NIntegrate[φ0[x] φ3[x], {x, 0, 1}]

-0.0138434

A04 = NIntegrate[φ0[x] φ4[x], {x, 0, 1}]

0.011854

A11 = NIntegrate[φ1[x] φ1[x], {x, 0, 1}]

0.490686

A12 = NIntegrate[φ1[x] φ2[x], {x, 0, 1}]

-0.0245773

A13 = NIntegrate[φ1[x] φ3[x], {x, 0, 1}]

0.0253637

A14 = NIntegrate[φ1[x] φ4[x], {x, 0, 1}]

-0.021995

A22 = NIntegrate[φ2[x] φ2[x], {x, 0, 1}]

0.50254

A23 = NIntegrate[φ2[x] φ3[x], {x, 0, 1}]

-0.0327288

A24 = NIntegrate[φ2[x] φ4[x], {x, 0, 1}]

0.0290354
```

```
A33 = NIntegrate[φ3[x] φ3[x], {x, 0, 1}]
```

```
0.506212
```

```
A34 = NIntegrate[φ3[x] φ4[x], {x, 0, 1}]
```

```
-0.0321677
```

```
A44 = A33 = NIntegrate[φ4[x] φ4[x], {x, 0, 1}]
```

```
0.502532
```

```
A = {
   {A00, A01, A02, A03, A04},
   {A01, A11, A12, A13, A14},
   {A02, A12, A22, A23, A24},
   {A03, A13, A23, A33, A34},
   {A04, A14, A24, A34, A44}
  };
```

```
A // MatrixForm
```

$$\begin{pmatrix} 0.477176 & -0.0107339 & 0.0135097 & -0.0138434 & 0.011854 \\ -0.0107339 & 0.490686 & -0.0245773 & 0.0253637 & -0.021995 \\ 0.0135097 & -0.0245773 & 0.50254 & -0.0327288 & 0.0290354 \\ -0.0138434 & 0.0253637 & -0.0327288 & 0.502532 & -0.0321677 \\ 0.011854 & -0.021995 & 0.0290354 & -0.0321677 & 0.502532 \end{pmatrix}$$

```
Det[A]
```

```
0.0291331
```

```
B0 = NIntegrate[f[x] φ0[x], {x, 0, 1}]
```

```
0.18102
```

```
B1 = NIntegrate[f[x] φ1[x], {x, 0, 1}]
```

```
-0.0438533
```

```
B2 = NIntegrate[f[x] φ2[x], {x, 0, 1}]
```

```
0.124863
```

```
B3 = NIntegrate[f[x] φ3[x], {x, 0, 1}]
```

```
0.116593
```

```
B4 = NIntegrate[f[x] φ4[x], {x, 0, 1}]
```

```
0.0900137
```

```
B = {B0, B1, B2, B3, B4};
```

```
X = LinearSolve[
   (*matrice dei coefficienti*)
   A,
   (*vettore dei termini noti*)
   B
 ]
```

```
{0.374471, -0.0754912, 0.242627, 0.272848, 0.17043}
```

```
S[x_] = X[[1]] * φ0[x] + X[[2]] * φ1[x] + X[[2]] * φ2[x] + X[[3]] * φ3[x] + X[[4]] * φ4[x]
```

```
0.374471 Sin[10 x] - 0.0754912 Sin[20 x] -
 0.0754912 Sin[30 x] + 0.242627 (1 + Sin[40 x]) + 0.272848 Sin[50 x]
```

```
Plot[
 {f[x], S[x]},
 {x, 0, 1},
 PlotRange → {-1.5, 1.5},
 PlotStyle →
  {
   {Thickness[0.003]},
   {Red, Thickness[0.0025]}
  },
 AxesLabel →
  {
   Style["x", Small, Red, Italic],
   Style["y", Small, Red, Italic]
  }
]
```